Name: ______________________ Student I.D.: ______________

Sample Exam 2 A

Physics 100, Spring 2007

Wednesday, March 16, 2007

Useful Equations and Numbers

Acceleration due to gravity...
on Earth = 10 m/s^2
on Mars = 3.7 m/s^2
on the Moon = 1.6 m/s^2

$$Impulse = Force \times Time$$

$$Work = Force \times Distance$$

$$Gravitational\ Potential\ Energy = mass \times (Acceleration\ of\ Gravity) \times Height$$

$$Kinetic\ Energy = \frac{1}{2}\ mass \times (speed)^2$$

$$Momentum = mass \times Velocity$$

$$Force = mass \times Acceleration$$

$$(force\ of\ gravity) = (mass) \times (acceleration\ due\ to\ gravity)$$

$$a^2 + b^2 = c^2$$

$20\ m/s = 45\ mph$	$1\ mile/minute = 60\ mph$
$1\ pound\ of\ force = 4.5\ Newton$	$1\ km = 0.6\ miles$
$\sqrt{3^2 + 4^2} = 5$	$1\ hour = 3,600\ seconds$
$1\ minute = 60\ seconds$	$1\ g = 10\ m/s^2$
$1\ m/s = 3.6\ km/hour$	$1\ Newton = 1\ \frac{kg\ m}{s^2}$
$1\ m = 3.2\ feet$	$1\ Joule = kg\ m^2/s^2$
$\sqrt{200} = 14.14$	$1\ Calorie = 1,000\ calories$
$1\ Calorie = 4,200\ J$	$\frac{1}{6} = 0.167$

DO NOT OPEN EXAM UNTIL INSTRUCTED TO DO SO!
TURN OFF YOUR CELL PHONE!

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Answer Key for Exam A

Section 1. **True/False (1 pts. each)**

False To throw a ball, you do not need to exert any impulse on it.

False Kinetic Energy, just like momentum, is a vector quantity and special care must be taken when adding kinetic energies together.

False It is possible for you to produce a net impulse on an automobile by sitting inside and pushing on the dashboard.

False Lifting a 50 kg sack a vertical distance of 2 m requires more work than lifting a 25 kg sack a vertical distance of 4 m.

False If something doubles its speed, both its momentum and kinetic energy double.

True The source of energy that we get when we eat food ultimately came from nuclear fusion occurring in the sun.

True Hydrogen promises to transfer energy in a clean-burning way, possibly eliminating smog and pollution generated by cars.

True Long-range cannons have long barrels because the force of the expanding gasses exerted on the shell (the ballistic) acts over a longer distance, increasing the amount of work done on the shell and thereby giving it more kinetic energy.

True The kinetic energy of a pendulum is at its maximum when it is at the lowest point of its swing.

False The gravitational potential energy of a pendulum is at its maximum when it is at the lowest point of its swing.

False An adult 5,000 kg elephant moving at 2 m/s has more kinetic energy than a younger 2,500 kg elephant moving at 5 m/s.

Section 2. **Multiple Choice (2 pts. each)**
Choose the single best answer unless instructed to do otherwise.

1. Which of the following are *vector* quantities? (Circle all that apply)
   - (a) Energy
   - **(b)** Momentum
   - (c) Speed
   - **(d)** Velocity
   - **(e)** Force

2. As you take this exam, you are sitting in a chair. What is the reaction to the force the chair is exerting on you?
   - (a) The force of gravity acting on you caused by the Earth
   - (b) The force of gravity acting on the Earth caused by you
   - (c) There is no reaction force in this case because you are not accelerating
   - **(d)** The force you are exerting on the chair

3. Why do padded dashboards make automobiles safer in collisions?
   - (a) Because they reduce the amount of impulse that is imparted on the passengers when they hit them
   - (b) Because they reduce the amount by which the passengers' momentum change
   - (c) Because they increase the stopping force
   - **(d)** None of the above

4. A punch with a bare fist is more powerful than with a boxing glove because:
   - (a) A bare fist can impart more impulse
   - **(b)** The glove and fist can impart the same impulse, but the glove will do so with a lower force for a longer time
   - (c) The punch with a glove is more powerful because it is larger and can provide more force than the fist
   - (d) None of the above are true

5. The potential energy of a skydiver falling at a *constant* velocity (i.e. her *terminal velocity*) is transformed into which of the following (circle all that apply):
   - (a) Kinetic Energy
   - **(b)** Thermal Energy (Heat)
   - **(c)** Noise
   - (d) Momentum
   - (e) This is a trick question: a skydiver falling with a *constant velocity* does not change potential energy

6. A chocolate glazed cake donut from Dunkin Donuts has nearly 300 Calories in it. This is the same amount of energy required to accelerate a $1,000\ kg$ car up to a speed of ______? (*Hints: (1) Ignore air resistance. (2)The conversions between different units are all correct.*)

   (a) $\sqrt{2 \times 0.3}\ m/s$, which is roughly 0.77 m/s or 1.7 mph

   (b) $\sqrt{2 \times 1,260,000}\ m/s$, which is roughly 1587 m/s, or 3,550 mph

   $\boxed{\text{(c)}}$ $\sqrt{2 \times 1,260}\ m/s$ which is roughly 50 m/s or 112 mph

   (d) None of the above

7. If gasoline has 42,000 J per gram, then a 2,000 Calorie per day diet is equivalent, in energy, how many grams of gasoline per day?

   (a) 21 grams of gasoline per day

   (b) 200 kg of gasoline per day

   (c) 20 grams of gasoline per day

   $\boxed{\text{(d)}}$ 200 grams of gasoline per day

Continue on to the next page...

Section 3. **Short Answer Questions (5 pts. each)**

8. A skydiver that weighs 600 N jumps out of an airplane. Just after opening her parachute, she experiences an upward resistive force of 2,000 N. **(A)** What is this upward resistive force called? **(B)** What is the magnitude and direction of her acceleration? **(C)** Is her speed increasing or decreasing?

   Answer: (A) Air resistance
   (B) The magnitude and direction of her acceleration are given by Newton's Second Law:

$$\vec{F}_{net} = mass \times \overrightarrow{accel.}$$

$$\overrightarrow{accel.} = \frac{\vec{F}_{net}}{mass}$$

We find $\vec{F}_{net}$ in the usual way:

$$\vec{F}_{net} = 600\ N\ (down) + 2,000\ N\ (up) = 1,400\ N\ (up)$$

Note: the '+' sign is the right one to use, since we're adding vectors. If we weren't keeping track of the directions (i.e. not writing out the (down) and (up), then we'd have to use a minus sign (which only works in cases like this where everything points in the same or completely opposite directions–1D motion). So long as you know the net force is up, then there wouldn't be any problems with your answer if you chose to work with magnitudes and minus signs.

Now we know the skydiver *weight* 600 N. Her weight is the same thing as the force she experience from gravity:

$$weight = mass \times (accel.\ due\ to\ gravity)$$

So her mass is

$$mass = \frac{weight}{accel.\ due\ to\ gravity} = \frac{600\ N}{10\ m/s^2} = 60\ kg$$

Now we just plug these things in to our expressiong for $\overrightarrow{accel.}$ to get the answer:

$$\overrightarrow{accel.} = \frac{1,400\ N\ (up)}{60\ kg} = 23.3\ m/s^2$$

Of course, the fractional expression, $1,400\ N\ (up)/60\ kg$ would be perfectly adequate.
(C) Air resistance, like dynamic friction, points in the direction opposite to the motion (i.e. air resistance points in the opposite direction as the velocity). Since the net force points in the same direction as the air resistance (namely up), then this is a case where the net force points in the opposite direction as the velocity. As we've discussed many times, this means that *the speed of the skydiver is decreasing.*

9. The ThrustSSC is a (near) supersonic car, which uses two jet engines. It has a mass of roughly 10,000 kg, and the two engines combined provide a thrust force of almost 100,000 N. How fast would the ThrustSCC be moving after traveling 5 m, if it started rest? *Ignore air resistance*[2].

[2] This note was not included on the original sample test– sorry about that! It is a practice test, after all! :)

Answer: This question gives us a force, a distance, and asks for a velocity. The force and distance part should clue you in to looking at work:

$$Work = Force_{\|} \times Distance$$

I am defining $F_{\|}$ to be the component of the force that points in the direction of motion[3]. In our case, the whole thrust force points in the same direction as the motion (velocity vector), so the work is simply given by:

$$Work_{thrusters} = 100,000\ N \times 5\ m = 500,000\ J$$

Now, the work-energy theorem tells us that a net work done on something changes its kinetic energy (KE:

$$Work = \Delta KE = KE_{final} - KE_{initial}$$

Since kinetic energy is defined as:

$$KE = \frac{1}{2} mass \times speed^2$$

We know that $KE_{initial} = 0$, since the ThrustSSC wasn't moving, originally. Therefore, in this case, the work energy theorem tells us that:

$$Work = 500,000\ J = \Delta KE = KE_{final} = \frac{1}{2} mass \times speed_{final}^2$$

Solving for the speed, we find:

$$speed_{final} = \sqrt{\frac{2 \times 500,000\ J}{10,000\ kg}} = \sqrt{100\ m^2/s^2} = 10\ m/s$$

Pretty good acceleration!

[3]The textbook writes the equation as $Work = Force \times Distance$, but then states that it's really the component of the force parallel to the motion that matters– I am simply cleaning up the equation a bit to reinforce that very important subtlety.

10. Figure 1 depicts a roller coaster. Ignoring friction and air resistance, **(A)** at which point is the roller coaster moving the fastest? **(B)** The slowest? **(C)** Which point does it have the maximum gravitational potential energy? **(D)** If the roller coaster weighs 2,000 N and has a speed of zero at Point A, how fast is it moving at Point E?

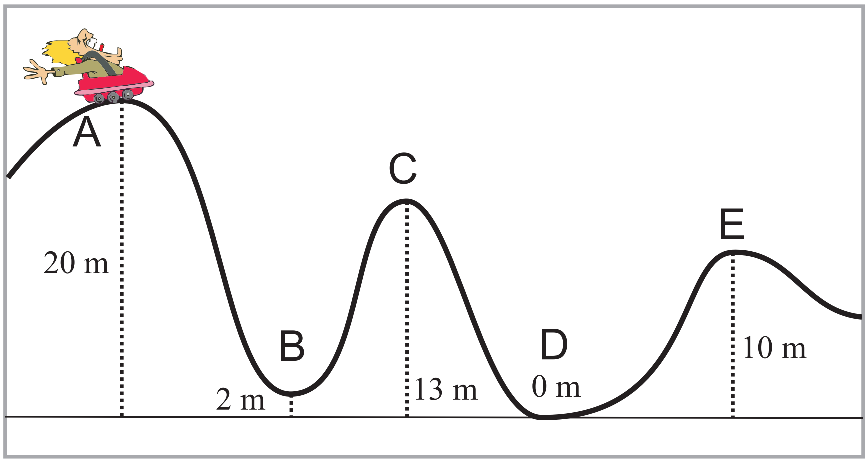

Figure 1: The depiction of the roller coaster in Problem 10

Answer: **(A)** The roller coaster is moving the fastest at the point where it has the minimum potential energy, because energy conservation tells us that at that point, it has the most kinetic energy (a loss in potential energy equates to a gain in kinetic energy, ignoring outside forces). Now, gravitational potential energy PE_{grav} is given by:

$$PE_{grav} = mass \times (accel.\ due\ to\ grav) \times height = weight \times height$$

Thus, the point where the roller coaster has the lowest PE_{grav}, and is therefore moving the fastest, is at point **D**.

(B) The point where the roller coaster is moving the slowest is where it has the most potential energy, and hence the least kinetic energy. The point where it has the most potential energy is **A**.

(C) The maximal gravitational potential energy is at **A**, for reasons explained in parts **(A)** and **(B)**.

(D) This part is a quantitative application of the conservation of energy principle. If the roller coaster starts at rest at point **A**, then it starts with a total energy, E_A, of:

$$E_A = PE_A + KE_A = (weight \times height_A) + (\frac{1}{2} mass \times speed_A^2)$$

$$E_A = ((2,000\ N) \times (20\ m)) + (0) = 40,000\ J$$

energy conservation tells us that at **E**, it must have the same total energy:

$$E_A = E_B = PE_B + KE_E = (weight \times height_E) + (\frac{1}{2} mass \times speed_E^2)$$

$$40,000\ J = ((2,000\ N) \times (10\ m)) + (\frac{1}{2} mass \times speed_E^2)$$

The mass of the roller coaster is found by dividing its weight ($2,000\ N$) by the acceleration due to gravity ($10\ m/s^2$). Doing this tells us that the roller coaster's mass is $200\ kg$:

$$40,000\ J = 20,000\ J + \frac{1}{2}(200\ kg) \times speed_E^2$$

Doing some math yields:

$$20,000\ J = (100\ kg) \times speed_E^2$$

$$speed_E = \sqrt{\frac{20,000\ J}{100\ kg}} = \sqrt{200\ m^2/s^2} = 14.1\ m/s$$

Where, again the expression $\sqrt{200\ m^2/s^2}$ would have been adequate for full credit.

11. Consider a sled with a cannon mounted on it sitting at rest on a sheet of ice, as shown in Figure 2. If the cannon fires snowballs with a mass of 1 kg each at a speed of 500 m/s, and the sled and cannon have a combined mass of 500 kg, about how many snowballs must the cannon fire for the sled to be moving at a speed of 10 m/s. *Hint: ignore friction, air resistance, and the change in mass of the cannon and sled due to the firing of the snowballs.*

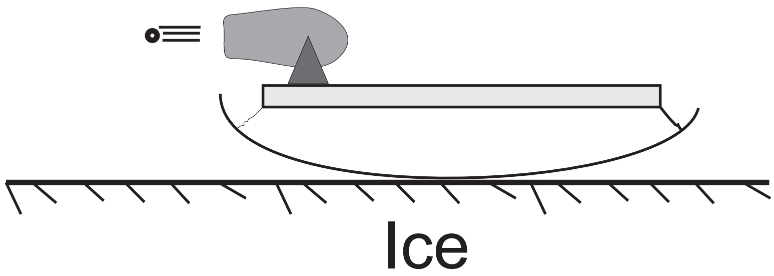

Figure 2: Problem 11: A snowball cannon mounted on a sled which is on a sheet of ice.

Answer: This is an impulse/conservation of momentum problem. During the firing of a snowball, each snowball gains the same amount of momentum, $P_{snowball}$[†], pointing backwards. To conserve momentum, then the sled must gain the same momentum, only in the forward direction.

This can also be thought of in terms of impulse: the snowball and sled exert the same force on each other, but in opposite directions (Newton's 3rd Law) for the same amount of time. Since impulse is the product of force times time, we know that the sled and the snowball therefore experience the same impulse, but in opposite directions. Impulse is also defined as a change of momentum, so we know that the snowball and sled undergo the same change of momentum, but in opposite directions.

So, we know that the firing of each snowball imparts the sled with an extra amount of momentum, $P_{snowball}$. So now all we have to do is figure out how much of a momentum change the sled must undergo to reach a speed of 10 m/s, divide that number by $P_{snowball}$ (which we must figure out), and that will be the number of snowballs fired.

Now, if the cannon will fire a 1 kg snowball out of its barrel at 500 m/s, then it imparts a momentum given by:

$$Impulse = \Delta momentum = (1\ kg) \times (500\ m/s) = 500\ kg\, m/s$$

This is really our $P_{snowball}$:

$$P_{snowball} = 500\ kg\, m/s$$

If the initial speed of the sled is zero, and the final speed we're interested in is 10 m/s, then the sled must undergo a change of momentum of:

$$\Delta P_{sled} = 500\ kg \times 10\ m/s = 5,000\ kg\, m/s$$

[†]Note: there is a bit of subtlety here: the snowballs leaving the sled won't actually leave with the same velocity, according to the lake's reference frame. Therefore, as the sled speeds up, the snowballs leave with less momentum according to the lake's reference frame. However, the firing cannon will still impart the same impulse on the snowball, and hence on the sled. So we are talking about changes in momentums– just like the astronaut-catch problem.

So the number of snowballs required is at least:

$$N_{snowballs} = \frac{\Delta P_{sled}}{P_{snowball}} = \frac{5,000\ kg\,m/s}{500\ kg\,m/s} = 10$$

12. Shown in Figure 3 is the collision between two cars. Just before the collision, one car, with a mass of m was headed North with a speed of $2v$. The other car, with a mass of $2m$, was headed East with a speed of v. When they collide, the two cars stick together. On the right-hand side of Figure 3, draw the resulting velocity vector that the two cars (stuck together) have after the collision.

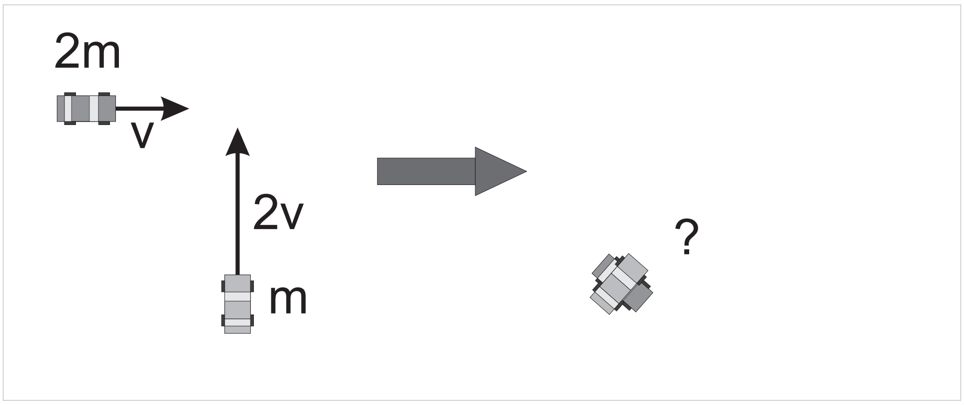

Figure 3: Problem 12: Two cars crash together.

Answer: Drawn on the diagram are velocities and masses. To solve this problem, we must deal with momenta. Yup, another conservation of momentum problem! What we have to do is find the momentum of each car. Momentum is defined as $mass \times velocity$, so we just have to multiply the provided masses times their velocities. It turns out, when you do this, that each car has the same magnitude of momentum– namely $2\,m\,v$! So the problem equates to adding two momenta, of the same magnitudes but at right angles to each other. We know that the resultant (i.e. vector sum) will point at 45°, and have a magnitude given by the Pythagorean Theorem ($a^2 + b^2 = c^2$)‡:

$$(2mv)^2 + (2mv)^2 = V_{final}^2$$

$$4\,m^2\,v^2 + 4\,m^2\,v^2 = V_{final}^2$$

$$V_{final} = \sqrt{8\,m^2\,v^2} = \sqrt{8}\,m\,v$$

The figure on the next page shows this result pictorially:

‡The Pythagorean Theorem has been covered in class and in the textbook.

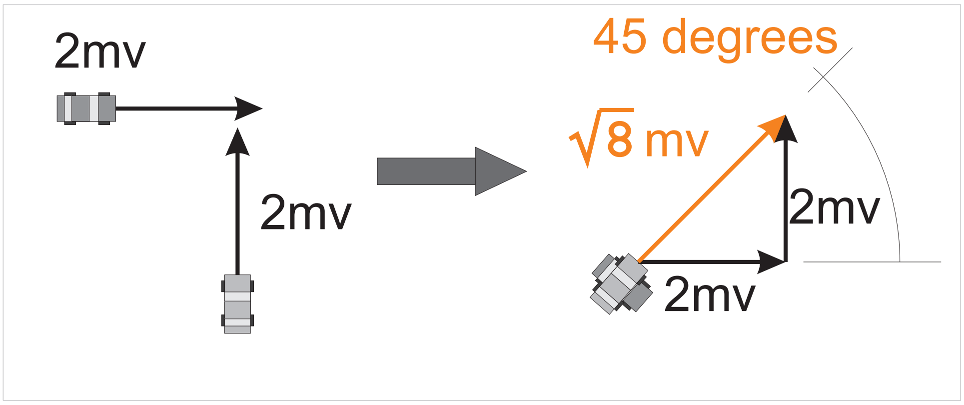

Figure 4: Problem 12: Two cars crash together. Momentum is conserved.

However, the problem is asking for the resulting velocity vector, not the momentum vector. To get the velocity vector, we divide the length of the momentum vector by the mass of the two cars stuck together: $3m$:

$$V_{final} = \frac{P_{final}}{m_{final}} = \frac{\sqrt{8}\, m\, v}{3\, m} = \frac{\sqrt{8}}{3}\, v$$

So the final diagram should look like the following:

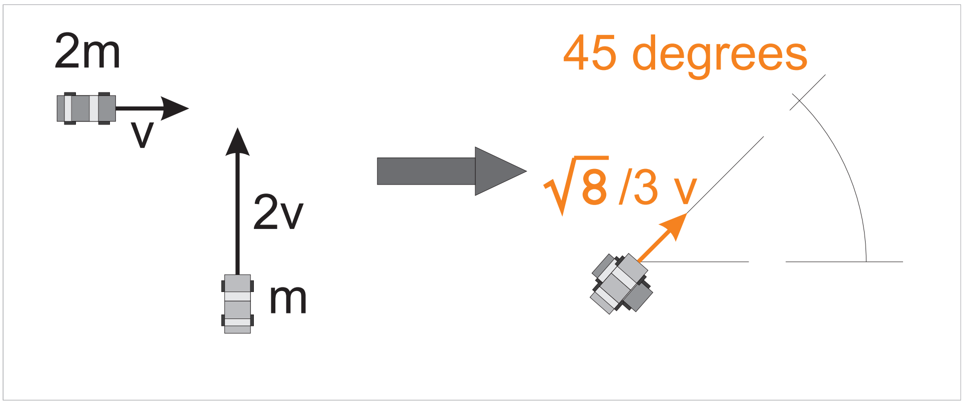

Figure 5: Problem 12: Two cars crash together and this is the final answer.

I have drawn the resulting velocity in scale with the original, noting that $\sqrt{8}/3 = 0.94$, meaning that the final velocity vector is just a little bit shorter than the original velocity vector representing a speed of v. Of course, on an exam, you are not expected to know this, so just drawing a vector at 45° and labeling its length as $\frac{\sqrt{8}}{3}\, v$ would have earned full points!